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Research Status Report #1

A Dual-Model Generalized Likelihood Ratio Approach to Self-Reorganizing Digital Flight Control System Design

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I. Introduction

In this report we summarize the results of the research efforts under Grant NSG-1112 through March 14, 1975. The personnel involved in this project up to this point are Prof. A.S. Willsky, Prof. N.R. Sandell, Dr. Keh-Ping Dunn, and graduate student E. Chow. It is anticipated that a second graduate student, Mr. R. Bueno, will joint the research effort in June 1975 to aid in the simulation and evaluation program.

The outline of this report is as follows. In the Sections II-VI we describe the work that has been done so far. Our efforts have been concentrated on the development of an analytical framework and a computation-simulation package for the GLR system. This framework will provide the basis for both analytical performance evaluation and for a simulation program. Both of these efforts will yield information on system performance limitations, sensitivity to parameter variations, and computationally efficient approximations. These issues, which will be considered in great detail during the remaining portion of the grant period, are described in Section VII.

II. Sensor and Actuator Jump and Step Failures-Open Loop Case

In the work done so far, we have concentrated on the development of detection algorithms for the four types of failure models described below. Computer algorithms have been developed, and the questions of computational simplification and performance evaluation are being considered with respect to these four models. The more complex problem of actuator and sensor gain changes and the question of failures that take the form of additional process or measurement noise have received some initial attention and will receive more in the near future (see Section VII).

It is felt that the detection mechanisms for these other failure modes will bear a resemblance to those for the four that have received the majority of our attention so far. Thus, the algorithms and performance analyses we have and will carry out for the basic four models will greatly aid the analyses for these other models.

The four basic failure models we have considered are (here x is n-dimensionals and z is m-dimensional

1. Actuator Step

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) + \sigma_{k+1,\theta}\nu \tag{1}$$

$$Z(k) = H(k)x(k) + v(k) \tag{2}$$

2. Actuator Jump

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) + \delta_{k+1,\theta}\nu \tag{3}$$

$$Z(k) = H(k)x(k) + v(k) \tag{4}$$

3. Sensor Step

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) \tag{5}$$

$$Z(k) = H(k)x(k) + v(k) + \sigma_{k,\theta}\nu \tag{6}$$

4. Sensor Jump

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) \tag{7}$$

$$Z(k) = H(k)x(k) + v(k) + \delta_{k,\theta}\nu \tag{8}$$

where w(k), v(k) are independent white zero mean Gaussian sequences with $E\{w(k)w'(k)\} = Q(k)$ and $E\{V(k)V(k)'\} = R(k)$,

$$\delta_{k,\theta} = \begin{cases} 1 & k=\theta \\ 0 & k\neq\theta \end{cases} \tag{9}$$

$$\sigma_{k,\theta} = \begin{cases} 1 & k\geq\theta \\ 0 & k<\theta \end{cases} \tag{10}$$

and ν is a constant vector of appropriate demension denoting the "abrupt change". The variable θ has the interpretation as the time of failure.

The motivation for these models is given in the proposal for this research [1] and in reference [2]. Note that the actuator jump model number 2 was considered in [2]. Note that we could have included a control term in these models. For example, we could have replaced (1) by

$$x(k+1) = \Phi(k+1,k)x(k) + B(k)u(k) + w(k) + \nu\sigma_{k+1,\theta} \tag{11}$$

We will consider this in the next section in which we discuss the closed loop case.

Suppose we design a Kalman filter (KF) for 1-4, where we assume that no failure occurs. The relevant equations are (see [1],[2] for details of the notation):

$$\hat{x}(k+1|k+1) = [I - K(k+1)H(k+1)]\Phi(k+1,k)\hat{x}(k|k) + K(k+1)Z(k+1) \tag{12}$$

$$K(k) = P(k|k-1)H'(k)V^{-1}(k) \tag{13}$$

$$P(k+1|k) = \Phi(k+1,k)P(k|k)\Phi'(k+1,k) + Q(k) \tag{14}$$

$$P(k|k) = P(x|k-1) - K(k)H(k)P(k|k-1) \tag{15}$$

$$V(k) = H(k)P(k|k-1)H^T(k) + R(k) \tag{16}$$

From studying these models we find that both the residual $\gamma(k) = Z(k) - H(k)\hat{x}(k|k-1)$ and the KF estimate $\hat{x}(k|k)$ can be decomposed into two parts:

$$\gamma(k) = \gamma_1(k) + \gamma_2(k) \tag{17}$$

$$\hat{x}(k|k) = \hat{x}_1(k|k) + \hat{x}_2(k|k) \tag{18}$$

when the variables with subscript 1 denote the residual and state estimate respectively when no failure has occurred and subscript 2 denote the "bias" developed in the KF due to failure. In addition, we find that in all cases

$$\gamma_2(k) = G(k,\theta)\nu \tag{19}$$

$$\hat{x}_2(k|k) = F(k,\theta)\nu \tag{20}$$

where G, F are functions of the system matrices, k, and θ only.

Now we are ready to apply the GLR method to determine the failure parameters θ and ν. We established two hypotheses:

H_0: no failure has occurred

H_1: failure has occurred at $\theta \leq k$

Then the GLR can be expressed as

$$\ell(k) = 2\ln \frac{P(\gamma(1),\ldots \gamma(k)|H_1,\ \theta=\hat{\theta}(k),\ \nu=\hat{\nu}(k))}{P(\gamma(1)\ \ldots\ \gamma(k)|H_0)} \tag{21}$$

$$= \sum_{j=1}^{k} \gamma'(j)V^{-1}(j)\gamma(j) - \sum_{j=1}^{k} [\gamma(j) - G(j;\hat{\theta}(k))\hat{\nu}(k)]'V^{-1}(j)\,[\gamma(j)-G(j);\hat{\theta}(k)\hat{\nu}(k)]$$

where $\hat{\theta}(k)$ $\hat{\nu}(k)$ are the MLE of θ,ν which maximizes $\ell(k)$. We have the decision rule:

$$\ell(k) \underset{H_0}{\overset{H_1}{\gtrless}} \varepsilon \tag{22}$$

where ε is some predetermined threshold.

It may be shown that $\hat{\nu}(k)$ is an explicit function of $\hat{\theta}(k)$:

$$\hat{\nu}(k) = C^{-1}(k;\hat{\theta}(k))d(k;\hat{\theta}(k)) \tag{23}$$

where

$$C(k;\theta) = \sum_{j=1}^{k} G'(j;\theta)V^{-1}(j)G(j;\theta) \tag{24}$$

$$d(k;\theta) = \sum_{j=1}^{k} G'(j;\theta)V^{-1}(j)\gamma(j) \tag{25}$$

Then $\hat{\theta}(k)$ is the value of $\theta \leq k$ that maximizes

$$\ell(k;\theta) = d'(k;\theta)C^{-1}(k;\theta)d(k;\theta) \tag{26}$$

The structural form of the GLR is given by (23)-(26) is the same <u>for all four types of failures</u>. The differences among the four are in the calculations of the matrices G and F. We now present the derivation of the equations for these matrices

1. A step in the state equation

$$x_2(k+1) = \Phi(k+1,k)x_2(k) + \sigma_{k+1,\theta}\nu \qquad , x_2(0) = 0 \tag{27}$$

$$Z_2(k) = H(k)x_2(k) \tag{28}$$

Thus

$$Z_2(k) = x_2(k) = 0 \qquad k<\theta \tag{29}$$

$$x_2(k) = \sum_{\ell=\theta-1}^{k-1} \Phi(k,\ell+1)\nu \qquad k\geq\theta \tag{30}$$

$$Z_2(k) = \sum_{\ell=\theta}^{k} H(k)\Phi(k,\ell)\nu \qquad k\geq\theta \tag{31}$$

Filter equation

$$\hat{x}_2(k|k) = \Theta(k,k-1)\hat{x}(k-1|k-1) + K(k)Z_2(k), \qquad \hat{x}(0|0)=0 \tag{32}$$

where

$$\Theta(k,k-1) = [I-K(k)H(k)]\Phi(k,k-1) \tag{33}$$

We then calculate

$$\hat{x}_2(0|0)=0 \qquad k<\theta \tag{34}$$

$$\hat{x}_2(k|k) = \sum_{j=0}^{k} \Theta(k,j)K(j)Z_2(j) \qquad k\geq\theta$$

$$= \sum_{j=0}^{k} \Theta(k,j)K(j) \sum_{\ell=0}^{j} H(j)\Phi(j,\ell)\nu$$

$$= \sum_{\ell=0}^{k} \sum_{j=0}^{k} \Theta(k,j)K(j)H(j)\Phi(j,\ell)\nu \tag{35}$$

Hence

$$\hat{x}_2(k|k) = F(k,\theta)\nu \tag{36}$$

$$F(k;\theta) = \begin{cases} 0 & k<\theta \\ \sum_{\ell=0}^{k} \sum_{j=0}^{k} \Theta(k,j)K(j)H(j)\Phi(j,\ell) & k\geq\theta \end{cases} \tag{37}$$

$$\gamma_2(k) = Z_2(k) - H(k)\Phi(k,k-1)\hat{x}_2(k-1|k-1)$$

$$= \sum_{\ell=0}^{k} H(k)\Phi(k,\ell)\nu - H(k)\Phi(k,k-1)F(k-1,\theta)\nu \qquad k\geq\theta \tag{38}$$

$$\gamma_2(k) = G(k,\theta)\nu \tag{39}$$

$$G(k;\theta) = \begin{cases} 0 & k<\theta \\ H(k)\,[\sum_{j=0}^{k} \Phi(k,j) - \Phi k,k-1)F(k-1;0)] & k\geq\theta \end{cases} \tag{40}$$

2. A jump in the state equation

$$x_2(k+1) = \Phi(k+1,k)x_2(k) + \delta_{k+1,\theta}\nu \tag{41}$$

$$Z_2(k) = H(k)x_2(k) \tag{42}$$

The result in 1 can easily be extended to this case

Noting that

$$x_2(k) = \Phi(k,\theta)\nu \tag{43}$$

and comparing this to (30), we see that we can obtain the desired equations by replacing

$$\sum_{\ell=\theta}^{k} \Phi(j,\ell)$$

with $\Phi(j,\theta)$. Hence

$$F(k;\theta) = \begin{cases} 0 & k<\theta \\ \sum_{j=0}^{k} \Theta(k,j)K(j)H(j)\Phi(j,\theta) & k\geq\theta \end{cases} \tag{44}$$

$$G(k;\theta) = \begin{cases} 0 & k<\theta \\ H(k)\ [\Phi(k,\theta)-\Phi(k,k-1)F(k-1;\theta)] & k\geq\theta \end{cases} \tag{45}$$

We will see that the similarity between these too cases allows us to make some algorithmic simplifications.

3. A step in the measurement equation

$$x_2(k+1) = \Phi(k+1,k)x_2(k) \qquad x_2(0)=0 \tag{46}$$

$$Z_2(k) = H(k)x_2(k) + \sigma_{k,\theta}\nu \tag{47}$$

Thus

$$x_2(k) = 0 \qquad \forall k \tag{48}$$

$$Z_2(k) = \begin{cases} 0 & k<\theta \\ \nu & k\geq\theta \end{cases} \tag{49}$$

Filter equations:

$$\hat{x}_2(k|k) = \Theta(k,k-1)\hat{x}_2(k-1|k-1) + K(k)Z_2(k) \quad ; \hat{x}_2(0|0)=0 \tag{50}$$

$$\hat{x}_2(0|0)=0 \qquad k<\theta \tag{51}$$

$$\hat{x}_2(k|k) = \sum_{j=0}^{k}\Theta(k,j)K(j)\nu \qquad k\geq\theta \tag{52}$$

Thus

$$F(k;\theta) = \begin{cases} 0 & k<\theta \\ \sum_{j=\theta}^{k} \Theta(k,j)K(j) & k\geq\theta \end{cases} \tag{53}$$

$$G(k;\theta) = \begin{cases} 0 & k<\theta \\ I & k=\theta \\ [I-H(k)\Phi(k,k-1)F(k-1;\theta)] & k>\theta \end{cases} \tag{54}$$

4. A jump in the measurement equation

$$x_2(k+1) = \Phi(k+1,k)x_2(k) \qquad x_2(0)=0 \tag{55}$$

$$z_2(k) = H(k)x_2(k) + \delta_{k,\theta}\nu \tag{56}$$

Similar to the case of a jump in the state, we note that $z_2(k) = \nu$ for $k=\theta$, otherwise $z_2(k)=0$. Then we have

$$F(k;\theta) = \begin{cases} 0 & k<\theta \\ \Theta(k,\theta)K(\theta) & k\geq\theta \end{cases} \tag{57}$$

$$G(k;\theta) = \begin{cases} 0 & k<\theta \\ I & k=\theta \\ -H(k)\Phi(k,k-1)F(k-1;\theta) & k\geq\theta \end{cases} \tag{58}$$

Finally, we note that the matrix G is essentially the only quantity that is needed in the implementation of the GLR detector. The matrix F is of importance in the implementation of a mechanism for compensation following detection. That is, the quantity

$$\hat{x}_2(k|k) = F(k;\theta)\nu \tag{59}$$

represents the response of the filter to the failure. On the other hand, the system response to the jump is of the form

$$x_2(k) = L(k;\theta)\nu \tag{60}$$

where we can specify L for the four cases (see below). In this case, after the detection of a failure, one might wish to correct the estimate via the equation

$$\hat{x}_{new}(k|k) = \hat{x}_{old}(k|k) + [L(k;\hat{\theta}(k))-F(k;\hat{\theta}(k))]\hat{\nu}(k) \tag{61}$$

One can also update the error covariance (to take into account our inaccurancies in estimating ν) by generalizing the result in [2]. We obtain

$$P(k|k)_{new} = P(k|k)_{old} + [L(k;\hat{\theta}(k))-F(k;\hat{\theta}(k))]C^{-1}(k;\hat{\theta}(k))[L(k;\hat{\theta}(k))-F(k;\hat{\theta}(k))] \tag{62}$$

The matrices F and L will also be useful in the considerations of compensating control action.

A straightforward calculation yields referring to the earlier derivations, we obtain

Case 1:

$$L(k;\theta) = \begin{cases} 0 & k<\theta \\ \displaystyle\sum_{\ell=\theta-1}^{k-1} \Phi(k,\ell+1) & k\geq\theta \end{cases} \tag{63}$$

Case 2:

$$L(k;\theta) = \begin{cases} 0 & k<0 \\ \Phi(k,\theta) & k>0 \end{cases} \tag{64}$$

Cases 3 and 4:

$$L(k,\theta) \equiv 0 \tag{65}$$

In the next section we consider the effect of feedback on this analysis.

III. The Closed Loop Case

Suppose the dynamics in our model includes a term of the form $B(k)u(k)$, and suppose we hypothesize a feedback law of the form

$$u(k) = T(k)\hat{x}(k|k) \tag{66}$$

One can show, in this case, that the filter residuals are unchanged, as the filter compensates for the effect of the feedback law. Thus, the matrix G is unchanged in all four cases. However, the matrices F and L are changed, as the effect of the jump is propagated from the system, through the filter, and back to the system again. The following analysis yields the desired equations in the closed loop case.

Consider the system equation (without failures)

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) + B(k)T(k)\hat{x}(k|k)$$

$$Z(k) = H(k)x(k) + v(k)$$

and the associated filter equations:

$$\begin{aligned}\hat{x}(k+1|k+1) &= [I-K(k+1)H(k+1)][\Phi(k+1,k) + B(k)T(k)]\hat{x}(k|k)\\ &\quad + K(k+1)\,[H(k+1)x(k+1) + v(k+1)]\\ &= \{[I-k(k+1)H(k+1)][\Phi(k+1,k)+B(k)T(k)]\\ &\quad + K(k+1)H(k+1)B(k)T(k)\}\,\hat{x}(k|k)\\ &\quad + K(k+1)H(k+1)\Phi(k+1,k)x(k) + K(k+1)H(k+1)w(k)+K(k+1)v(k+1)\end{aligned} \tag{67}$$

Combining the state and filter equations:

$$\begin{bmatrix} x(k+1) \\ \hat{x}(k+1|k+1) \end{bmatrix} =$$

$$\begin{bmatrix} \Phi(k+1,k) & B(k)T(k) \\ K(k+1)H(k+1)\Phi(k+1,k) & [I-K(k+1)H(k+1)][\Phi(k+1,k)+B(k)T(k)]+K(k+1)H(k+1)B(k)T(k) \end{bmatrix}$$

$$\times \begin{bmatrix} x(k) \\ \hat{x}(k|k) \end{bmatrix} + \begin{bmatrix} I \\ K(k+1)H(k+1) \end{bmatrix} \begin{matrix} 0 \\ K(k+1) \end{matrix} \begin{bmatrix} w(k) \\ v(k+1) \end{bmatrix} \tag{68}$$

with the abreviation

$$X(k+1) = A(k+1,k)X(k) + \Gamma(k)W(k) \tag{69}$$

Note that $X(k)$ is of dimension $2n$.

The four different cases of failure may be modeled as:

1. A step in the state

$$X(k+1) = A(k+1,k)X(k) + \Gamma(k)\ W(k) + \begin{bmatrix} \sigma_{k+1,\theta}\nu_{x} \\ 0 \end{bmatrix} \tag{70}$$

2. A jump in the state

$$X(k+1) = A(k+1,k)X(k) + \Gamma(k)\ W(k) + \begin{bmatrix} \sigma_{k+1,\theta}\nu_{x} \\ 0 \end{bmatrix} \tag{71}$$

3. A step in the measurement

$$X(k+1) = A(k+1,k)X(k) + \Gamma(k)\ W(k) + \begin{bmatrix} 0 \\ \sigma_{k,\theta}\nu_{m} \end{bmatrix} \tag{72}$$

4. A jump in the measurement

$$X(k+1) = A(k+1,k)X(k) + \Gamma(k)\ W(k) + \begin{bmatrix} 0 \\ \delta_{k,\theta}\nu_{m} \end{bmatrix} \tag{73}$$

where ν_x is the n dimensional "failure vector" for actuator failures and ν_m is the m dimensional "failure vector" for sensor failures. Separating the effect of failure from noise, we have,

$$X_2(k+1) = A(k+1,k)X_2(k) + \Gamma(k)D \quad ; \qquad X_2(0)=0 \tag{74}$$

where D is any of the four partiotioned "failure vectors". Then

$$\gamma_2(k) = Z_2(k) - \hat{Z}_2(k)$$

$$\gamma_2(k) = H(k)\{[I\ 0]X_2(k) - (\Phi(k,k-1) + B(k-1)T(k-1))\ [0\ I]\ X_2(k-1)\} \quad (75)$$

$$\text{where} \quad X_2(k) = \sum_{j=\theta-1}^{k-1} A(k,j+1)\Gamma(j)D \quad (76)$$

It may be shown that $\hat{X}_2(k|k)$ and $\gamma_2(k)$ have the form:

$$\hat{X}_2(k|k) = F(k;\theta)\nu$$

$$\gamma(k) = G(k;\theta)\nu$$

where dimension of ν dependends on the type of failure.

IV. Computational Considerations

We note that in general the GLR detector requires a growing bank of filters -- i.e. we must check for all values of θ from 0 to the present time. For practical situations, we may restrict attention to "data windows" -- i.e. at time k we only check values of θ that satisfy

$$k - M \leq \theta \leq k \quad (77)$$

We note that this still requires a great deal of computation, as we must store the last M residuals and must implement at least one matched filter, requiring a state of dimension (M+1)m (i.e., we must calculate equation (25)).

One possibility, described in the proposal [1], involves the use of the WSSR technique (see [1] and [3]), in which we merely square and sum the residuals. We have since developed a variant of the GLR which requires essentially the same computational effort as the WSSR and which we feel may prove to be an extremely useful detection tool. Suppose we assume that $\nu=\nu_0$ is known. Then, the GLR reduces to

$$\ell(k;\theta) = \sum_{j=\theta}^{k} [2\gamma(j) - G(j;\theta)\nu_0]'V^{-1}(j)G(j;\theta)\nu_0 \tag{78}$$

which is strikingly similar to the WSSR. If further state that at any time we will consider only

$$\hat{\theta}(k) = k-N \tag{79}$$

we remove the optimization over θ and further reduce the burden. Note that we may wish to compute several different ℓ's for different ν_0's and different models (i.e. 1-4). In this way, although we will not directly obtain an estimate of ν, we can obtain failure isolation information, that is not available with the WSSR, with relatively little computation. We propose to evaluate the usefulness of this approach, and describe our proposed research in this direction in Section VII.

We note that a major computational simplification occurs if the system of interest and its associated filter are time-invariant (i.e. the filter is in steady-state). In this case

$$\begin{aligned} G(k;\theta) &= G(k-\theta) \\ F(k;\theta) &= F(k-\theta) \\ L(k;\theta) &= L(k-\theta) \\ C(k;\theta) &= C(k-\theta) \end{aligned} \tag{80}$$

which eases the computational algorithm for determining the GLR equations and greatly reduces the necessary on-line storage. It is this case that has been considered in the computer algorithms described in Section VI.

Finally, we note that the GLR, even if we consider fixing $\hat{\theta} = k-M+1$, requires a filter of dimension Mm. This is basically due to the limited memory mature of the filter (equation (25)). It may be possible, by considering age-weighted filters, to greatly reduce this dimension. This question will also be considered in the future (see Section VII).

V. Othe Failure Models

There are basically four other failure models to be considered.

5. Added Actuator Noise

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) + \xi(k)\sigma_{k+1,\theta} \tag{81}$$

$$z(k) = H(k)x(k) + v(k) \tag{82}$$

Here ξ is an additional white noise process.

6. Added Sensor Noise

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) \tag{83}$$

$$z(k) = H(k)x(k) + v(k) + \xi(k)\sigma_{k,\theta} \tag{84}$$

7. Change in Actuator Gain

$$x(k+1) = \Phi(k+1,k)x(k) + B(k)u(k) + w(k) + Mu(k)\sigma_{k+1,\theta} \tag{85}$$

$$z(k) = H(k)x(k) + v(k) \tag{86}$$

8. Change in Sensor Gain

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) \tag{87}$$

$$z(k) = H(k)x(k) + v(k) + Jx(k)\sigma_{k,\theta} \tag{88}$$

We note that we can also consider the closed loop case, but, as discussed in Section III, there is no difference if one considers the question of detection only. Closed loop analysis is needed, however, when one wants to consider the question of compensation.

We have performed some initial analyses for these models and we briefly discuss them here. We note first, however, that it is quite possible that the GLR detectors for models 1-4 will be able to detect the failures represented in models 5-8. We plan to investigate this possibility, as it will lead to an overall reduction in detector complexity (see Section VII).

For models 5 and 6, one can show that $\gamma_2(k)$ is a zero mean random variable with precomputable covariance, which is a function of the covariance of $\xi(k)$ (the exact equations will be reported in the next report). Note that a crucial point is that the $\gamma_2(k)$ are not white, and the GLR technique in this case essentially examines the correlation behavior of the residuals. We also point out that the full GLR requires the estimation of the covariance of ξ, which can be accomplished with the aid of techniques such as those of Mehra [4]. If we hypothesize a fixed value for this covariance, the required computations become far simpler.

The sensor gain model #8 leads to a nonlinear estimation-detection problem. If we assume that J is unknown, we have a nonlinear estimation problem (the product of J and x causes the problem). If we assume that J is known (e.g., J is all zero except one row which is the negative of the corresponding row of H-- this is the "hard-over" failure model), one obtains

$$\gamma_2(k) = \sum_{j=\theta}^{k} \Lambda(k,j;\theta)x(k) \tag{89}$$

where the Λ can be precomputed (the precise equations will be reported in the next progress report). In this case, the optimal GLR would require <u>the estimation of x</u>. It is possible to avoid this if we make an approximation. Rewriting the sensor equation, we have

$$z(k) = H(k)x(k) + v(k) + J\hat{x}(k|k)\sigma_{k+1,\theta} + J(x(k)-\hat{x}(k|k)\sigma_{k+1,\theta}$$

$$= H(k)x(k) + v(k) + J\hat{x}(k|k)\sigma_{k+1,\theta} + Je(k)\sigma_{k+1,\theta}$$

where e is the error in the estimate $\hat{x}(k|k)$. As we can precompute the statistic of e, our model is of the approximate form

$$z(k) = H(k)x(k) + v(k) + [J\hat{x}(k|k) + \xi(k)]\sigma_{k+1,\theta} \tag{90}$$

We have briefly discussed how one can handle the $\xi\sigma_{k+1,\theta}$ term in the consideration of model 6. The term $J\hat{x}(k|k)\sigma_{k+1,\theta}$ represents a step failure with the size of the step <u>modulated by the estimated $\hat{x}$, which is known</u>.

The detection of such a phenomenon involves problems similar to those encountered in model 7 (although model 8 is somewhat easier, as all sensor failure analyses are slightly more straightforward than the corresponding actuator analyses). As we have performed some analysis for the actuator model 7, we will concentrate our present discussion on it. Full analyses of these models will be given in the next report. Consider model 7. Note that the failure in this case is a step jump modulated by the known inputs u(k). This type of failure is actually an extension of the failure of model 2 (the actuator jump with $\nu(j) = M(j)u(j)$, $\theta-1\leq j\leq k$) as follows:

$$x(k+1) = \Phi(k+1,k)x(k) + B(k)u(k) + w(k) + \sum_{j=\theta-1}^{k} M(j)u(j)\delta_{k+1,j+1} \tag{91}$$

Using the results we have obtained in Section II and applying superposition principle, we have

$$\hat{x}_2(k|k) = \sum_{j=\theta-1}^{k} F(k;j+1)M(j)u(j) = \sum_{j=\theta-1}^{k-1} F(k;j+1)M(j)u(j) \tag{92}$$

and

$$\gamma_2(k) = \sum_{j=\theta-1}^{k} G(k;j+1)M(j)u(j) = \sum_{j=\theta-1}^{k-1} G(k;j+1)M(j)u(j) \tag{93}$$

where $F(k;j)$ and $G(k;j)$ are given by (44) and (45), respectively, and where we have used the fact that $F(k;j)=0$ and $G(k;j)=0$ when $k<j$.

We have obtained the expressions for $\hat{x}_2(k|k)$ and $\gamma_2(k)$, but determining the MLE's of θ and $M(j)$, $\theta-1\leq j\leq k$ which maximize $\ell(k)$ of (21) is not an easy task. The control inputs $u(k)$ dependence complicates the whole derivation. So far we are able to obtain an expression for $\hat{M}(j|\theta;k)$ (MLE of $M(j)$ given observations up to time k and knowing jump occurs at time θ) with the following assumptions:

(i) $u(k)$ is a scalar.

(ii) the matrix $H'(k)V^{-1}(k)H(k)$ is invertable for all k.

These assumptions can be relaxed and will be discussed in the next report. Some discussion on these are also given in Section VII. We will only give some results on time varying case to demonstrate the complexity of this problem. The complete derivations and discussions of this model will be reported in the future. For the sake of simplicity, let us denote

$$\pi(k) = [H'(k)V^{-1}(k)H(k)]^{-1}[H'(k)V^{-1}(k)] \tag{94}$$

$$G(k;\theta) = H(k)\Psi(k;\theta) \tag{95}$$

and therefore

$$\pi(k)G(k;\theta) = \Psi(k;\theta) \tag{96}$$

After a nontrivial derivation, we have

$$\hat{M}(k-1|k;k) = \pi(k)\gamma(k)|u(k-1) \tag{97}$$

$$\hat{M}(k-2|k-1;k) = \hat{M}(k-2|k-1;k-1) \tag{98}$$

$$\hat{M}(k-1|k-1;k) = \hat{M}(k-1|k;k) - \Psi(k;k-1)\hat{M}(k-2|k-1;k-1)\frac{u(k-2)}{u(k-1)} \tag{99}$$

and furthermore

$$\hat{M}(k-i|k-j;k) = \hat{M}(k-i|k-j;k-j) - \sum_{\ell=k-j}^{k-i}\Psi(k-i+1;\ell)\hat{M}(\ell-1|\ell,\ell)\frac{u(\ell)}{u(k-i)} \qquad 1 \leq i \leq j \tag{100}$$

Contradicting to our intuition, the time invariant case is not as easy as we have seen in the other models. Further analyses of failure of this type are undertaken. A neat recursive formula for $\hat{M}(j|\theta;k)$, $\theta-1 \leq j \leq k-1$ and the use of the existing computer subroutines (see next section) to this problem are also under investigation.

VI. Computer Routines

In preparing the FORTRAN subroutines, we have taken the system to be time invariant and the KF to be in study state. Hence we have constant system matrices Φ, H, K, T. In addition the dependence of G, F, C, on k and θ becomes dependence on $\ell = k-\theta$ only. A list of available subroutine and brief descriptions are as follows:

A. XGFOL, MGFOL compute and store the G and F matrices for a data window of width M+1 for failures in the state and measurement respectively in the open loop case. A flag (SJ) determine whether the failure is a step (1.0) or a jump (0.0). Upon impecting the functional form of F, we find them cumbersome to evaluate. A simple approach is to use the fact that

$$F(k+1;\theta) = \Phi F(k;\theta) + KG(k+1,\theta)$$

Together with the expression for G, we have a pair of recursive expressive that can be easily implimented on a computer.

The time invariant equations to be implemented.

1. A step in the state

$$G(\ell) = \begin{cases} 0 & \ell<0 \\ H \sum_{j=\theta}^{\ell} \Phi^{j} - \Phi F(\ell-1) & \ell>0 \end{cases}$$

$$F(\ell) = \Phi F(\ell-1) + KG(\ell)$$

$$F(0) = KH$$

$$G(0) = H$$

2. A jump in the state

$$G(\ell) = \begin{cases} 0 & \ell<0 \\ H[\Phi^{\ell} - \Phi F(\ell-1)] & \ell\geq 0 \end{cases}$$

$$F(\ell) = \Phi F(\ell-1) + KG(\ell)$$

$$F(0) = KH$$

$$G(0) = H$$

Note that $\sum_{j=0}^{\ell}\Phi^{j} = \underbrace{\Phi(\Phi\ldots\Phi}_{\ell\Phi\text{'s}}(\Phi+I)+I\ldots+I)$.

If we set all the I's in the equation to 0 we get Φ^{ℓ}. Apart from the

difference of Φ^{ℓ} and $\sum_{j=0}^{\ell}\Phi^{\ell}$ in the expression for $G(\ell)$. The equations

for 1 and 2 are the same. Hence the flag SJ is used to set I to be the

identity or 0 matrix.

3. A step in the measurement

$$F(\ell) = \sum_{j=0}^{\ell} \{[I-KH]\Phi\}^{j}K \qquad \ell\geq 0$$

$$G(0)=I$$

$$G(\ell) = [I-H\Phi F(\ell-1)] \qquad \ell>0$$

4. A jump in the measurement

$$F(\ell) = \{[I-KH]\Phi\}^{\ell} K \qquad \ell\geq 0$$

$$G(0)=I$$

$$G(\ell) = -H\Phi F(\ell-1) \qquad \ell>0$$

Again, we use the same method to obtain one routine for both of the cases. The I's in the computation of $\sum_{j=1}^{\ell}[(I-KH)\Phi]^{j}$ and $G(\ell)$ in the step case can be set to 0 in order to obtain the equations in 4.

B. GFCL computes and stores the G & F matrices in the closed loop case. A flag LTF is used to indicates whether the failure is a jump in the state (LTF = 1), a step in the state (2), a jump in the measurement (3) or a step in the measurement (4). This routine does not employ the 2n dimensional expressions for the system X. Instead, it just use the basic equations of the state and filter

$$x_2(k+1) = \Phi x_2(k) + BT\hat{x}_2(k|k) + D_1$$

$$\hat{x}_2(k+1|k+1) = \Phi\hat{x}(k) + K\,H[\Phi x(k+1)-(\Phi+BT)\hat{x}_2(k|k) + D_2$$

where D_1 and D_2 are the failures and are never non zero simultaneously. We know that

$$x_2(\ell) = F1(\ell)\nu$$

$$x_2(\ell) = F(\ell)\nu$$

where ν is the failure of appropriate dimension.

Then

$$F1(\ell+1) = \Phi F1(\ell) + BTF(\ell) + I_1$$

$$F(\ell+1) = \Phi F(\ell) + KG(\ell+1) + I_2$$

$$G(\ell+1) = H[\Phi F1(\ell+1) - (\Phi+BT)F(\ell)]$$

where I_1, I_2 are identity matrices at least one of which is always forced to zero. For LTF = 1, $I_2=0$, and $I_1\neq 0$ only for $\ell=0$ indicating a jump in state; LTF=2, $I_2=0$ and $I_1=I\ \forall\ell\geq 0$, LTF=3, $I_1=0$, $I_2\neq 0$, only for $\ell=0$; LTF=4, $I_1=0$, $I_2=I\ \forall\ell\geq 0$. (Note that in this case F1 corresponds to the matrix L defined in Section II). Also the initial conditions for failures in state and measurement are different state

$F1(0)=I$

$F(0)=KH$

$G(0)=H$

measurement: $F1(0)=0$

$F(0)=K$

$G(0)=I.$

C. GC1NV, GTV1NV compute and store

$C^{-1}(\ell)$ and $G'(\ell)V^{-1}$ in a very straightforward manner.

$$C(\ell+1) = G'(\ell+1)V^{-1}G(\ell+1) + C(\ell) \qquad \ell>0$$

$$C(0) = G^{1}(0)V^{-1}G(0)$$

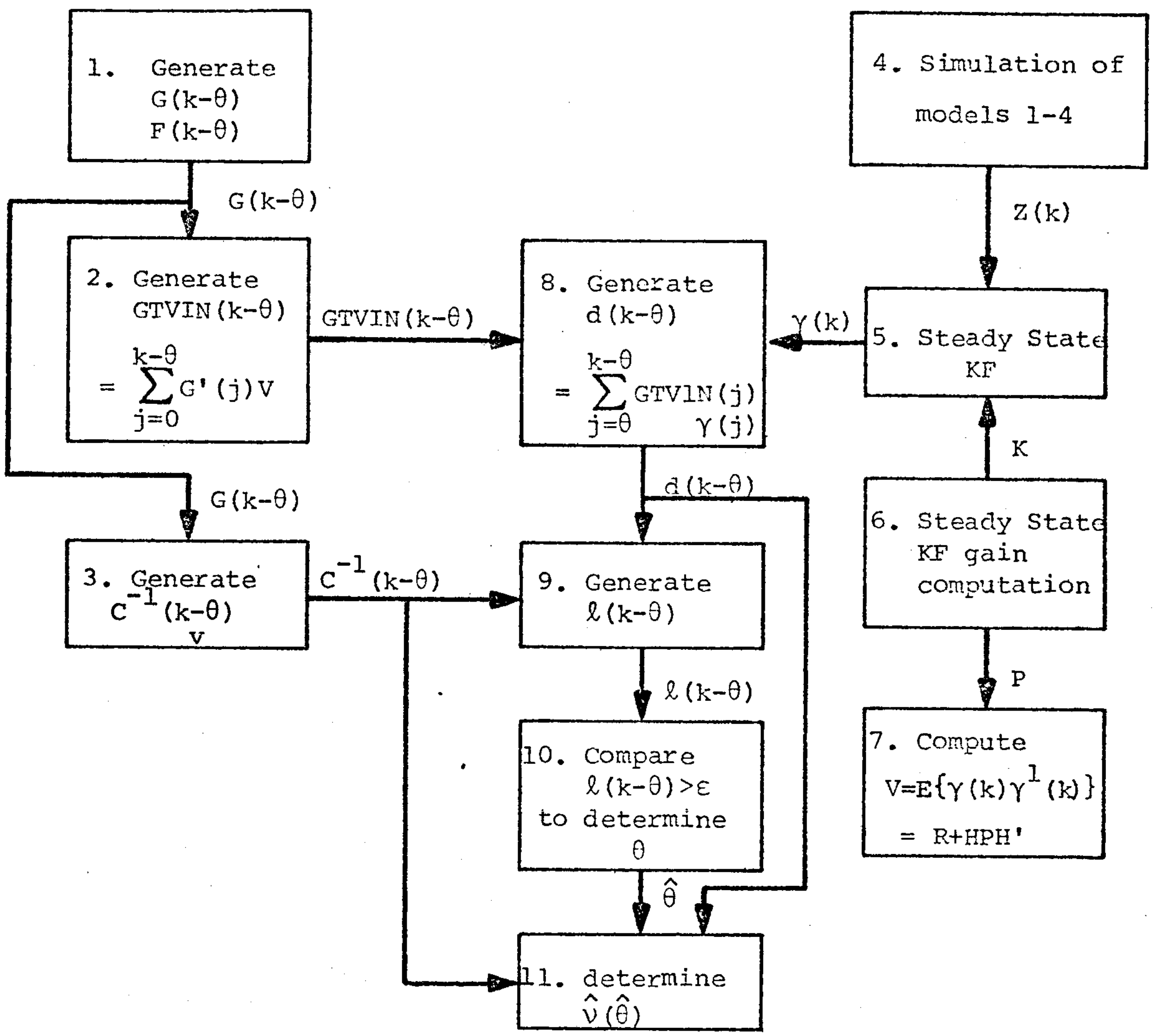
D. GEND, GENLOR computes and store the vectors $d(\ell)$ and GLR's respectively.

E. STOFCH, MADSUB, MINIT are support subroutine. STOFCH stores a matrix into a big storage or retrieve a matrix from storage, MADSUB adds or subtracts matrices. MINIT initializes matrices to 0 or wI when w is a real constant.

The simulation

As an initial attempt to examine the effectiveness of the GLR detection scheme, we have begun to assemble a simulation package as outlined in the flow diagram. Routines have been developed for steps 1,2,3,8, and 9. There are other computer packages at MIT that can easily be modified to accomplish steps 4,5,6 and 7. Steps 10 and 11 are being worked on. Failure models 1-4 will be examined in the simulation. It is anticipated that, together with the analysis of detection performance, the simulation will give useful insights into the practical considerations of the GLR detector implementation such as the data windows width M and the likelihood ratio threshold ε. Models 5-8 will also be included in the simulation as the analysis is completed.

Simulation Flow Diagram of GLR detection over a data window: $k-M \leq \theta \leq k$



VII. Work To Be Done

In this section we describe the questions we feel should be considered. Some of these are being considered and will be considered during the remainder of this grant period.

1. GLR Development

We plan to complete our development of the GLR equations for the 8 cases described in the preceding sections. This includes

(a) Completion of the analysis for models 5-8 (Section V). This involves the consideration of the full GLR problem, and the various simplifications that one obtains by considering fixed assumed jump sizes and fixed jump directions (such as changes in single columns of Band rows of H, corresponding to failures in particular actuators and sensors).

(b) Development of a complete computer package for the computation of the GLR equations for the various cases for systems of dimension up to 10. This includes the full GLR equations and the equations for the various simplifications developed in (a). We will also include a simulation section in the overall package. This section will be relatively easy to assemble, as subroutines developed for other projects at M.I.T. (including the MMAC work for the F-8 under Grant NSG-1018) can be directly adapted to the GLR package.

2. Analytical Study of Detection Performance

Some initial results presented in [2] indicate that a methodology can be developed for the calculation of probability P_D of correct detection and the probability P_F of false alarm. The probability P_F, defined by

$$P_D(\nu) = \int_{\varepsilon}^{\infty} p(\ell|H_1,\nu)d\ell$$

can be computed from tables, as ℓ is a non-central Chi-squared random variable.

We plan to carry this analysis out in detail and to consider the problem for the other four cases. The purpose of this study is to obtain data on the tradeoff among the various system parameters -- the threshold ε, the window length M+1, and the size of the failures that can be detected (i.e. we will examine $P_D(\nu)$ as a function of ν). Another important parameter is the delay time in detection. Let T be the number of time steps following a failure before it is detected. We can readily derive the following equations for the distribution of T

$$P(T=0) = P(\ell(\theta,\theta)>\varepsilon) \tag{101}$$

$$P(T\leq t) = P(T\leq t-1) + P(\ell(\theta+t,\theta)>\varepsilon|T\geq t) \tag{102}$$

Assuming we hypothesize a specific value for the failure, we can compute P(T=0), and thus the problem reduces to computing the conditional probability in (102). We plan to investigate the calculation of this quantity.

These probability calculations can also be used to obtain useful information concerning simplifications of the GLR. For example, suppose ℓ_1 and ℓ_2 are log-likelihood ratios for two different failure hypotheses. A useful quantity is the "cross-detection probability"

$$P_{12} = \int_{\varepsilon}^{\infty} p(\ell_1|H_2)d\ell_1$$

That is, P_{12} is the probability that one GLR detector will detect a failure corresponding to a second hypothesis. By examining such probabilities, we hope to obtain information as to the elimination of certain failure hypotheses which can be adequately detected by other detectors. In addition, this analysis will provide useful information concerning the performance of the computationally simpler fixed size GLR system.

Much of this analysis cannot be performed in the vacuum of general models. Therefore, we expect to consider several simple, low order examples that capture the basic properties of problem. Such analysis should provide guidelines for GLR design for higher order systems.

3. Simulation Studies

We plan to develop a low-order dynamical system which can be used to study the properties of the GLR system. Such a study, combined with the performance analysis described above, will yield desired information on the performance limitations of the GLR. The use of one basic model for a variety of tests will provide us with a common basis for the comparison of the various GLR systems and any approximations that are developed.

We eventually plan to apply the GLR to the F-8 aircraft (or another aerospace vehicle, such as the space shuttle or a control-configured vehicle); however, it is not clear that such a model would be appropriate for these initial tests. The resolution of the question of the model for these initial tests will be made in consultation with NASA-Langley personnel.

4. Sensitivity Analysis

We propose to study the effect of parameter errors on GLR performance. Some initial analysis has been performed indicating that sensitivity equations can be derived. We intend to pursue this analysis as far as possible. In parallel with this study, we will perform a series of simulation runs (using the same simple model discussed earlier) to obtain numerical data relating to the parameter sensitivity of GLR performance.

5. Compensation and Reorganization

In Section II we described a possible algorithm for the compensation of the Kalman Filter following the detection of a failure. We propose to develop analogous methods for the other failure models and to investigate the qualitative behavior of the overall system (i.e. does this feedback compensation add a destabilizing feature to the overall system). In addition, we will have to consider compensation methods for the simplified GLR algorithm which does not provide an estimate of the size of the failure. One possibility is to utilize a dual-mode procedure, in which, following detection by the simplified GLR, we switch to the full GLR for isolation and compensation. A second possibility is to develop a set of compensation rules to be utilized directly after detection by the simplified GLR. Again, these questions will be studied both via analysis and simulation.

We note that the type of compensation discussed in Section II deals basically with filter compensation, with system compensation being accomplished only through the feedback law $u=T\hat{x}$. In many cases (particularly those involving actuator failures) we will want to take more direct compensatory action. Several ideas are discussed in the proposal [1]. We propose to study these compensation methods in detail.

6. Computational Simplifications

Several possible computational simplifications have been discussed already. These include the limited optimization over θ (avoiding the growing dimensionality problem), the steady-state algorithm (leading to time-invariant GLR equations), and the fixed jump size and direction algorithm (eliminating the estimation of the failure size). As mentioned in Section IV, another possible simplification would be to find a lower order approximate realization of the GLR algorithm. For example, the GLR algorithm for models 1-4 takes the form (in the time-invariant case)

$$d(k;\theta) = \sum_{j=\theta}^{k} G'(j-\theta)V^{-1}\gamma(j)$$

The realization problem is to find four matrices A,C,E, and N such that

$$d(k;\theta) = \sum_{j=0}^{k-1} CA^{j-k+1}E\, V^{-1}\gamma(j) + N\gamma(k)$$

The GLR implementation described in Section II-VI provides an exact realization. However, the limited memory nature of the exact algorithm requires a high-dimensional realization. Thus, it seems appropriate to seek lower order approximations to the exact GLR. In filtering theory, age-weighted filters often behave in a similar manner to that observed in limited memory filters. Thus, we propose to consider some type of age-weighted lower-order approximation. We plan to carry the analysis of such approximations as far as possible, both through theoretical developments and simulation.

7. Several Additional Questions

There are several additional questions that we would like to consider:

(a) The GLR as developed is based on the use of the innovations sequence

$$\gamma(k) = z(k) - H(k)\hat{x}(k|k-1)$$

Is there any benefit in using the residuals defined by the equation

$$\gamma(k) = z(k) - H(k)\hat{x}(k|k)$$

(b) Can we develop a GLR algorithm for jumps in the transition matrix. That is, consider the detection of changes in dynamics of the form

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) + \Psi x(k)\sigma_{k+1,\theta}$$

It should be noted that there are several similarities between this model and models 7 and 8 described in Section V. We also note that detection of changes in Φ and B (the actuator gain) can be interpreted as detection of changes in operating conditions.

(c) The question of system observability must be addressed. The implementation of the GLR requires the inversion of the matrix $C(k;\theta)$. If this inverse does not exist, we are presently using a pseudo-inverse instead. However, we note that C(k;) can be interpreted as the information matrix describing what information about a failure at time θ is present in the measurements $z(\theta),\ldots, z(k)$. Thus it may be more desirable to use a different "pseudo-inverse" for C that takes into account our a priori knowledge about likely failure modes and the observability behavior for each of them.

8. Application of the GLR to an Aerospace Application

We plan to apply the techniques we have developed to problem of the digital control of an aerospace vehicle (e.g. the F-8 or space shuttle). We propose to begin this part of the study by examining several flight conditions in detail in order to obtain information on the parameter and detector complexity tradeoffs for this problem. It is our eventual aim to obtain a design procedure over the full flight envelope and to consider the effects of the true nonlinear model on detector performance.

Problems 1-3 are being and will be considered in the near future. Problems 4 and 6 will be considered subsequent to the resolving of some of the questions in 1-3. Problems 5 and 8 are somewhat more long-range in nature, although simulations of the F-8 will be begun in the near future. The questions in problem 7 are somewhat peripheral and will be considered as time allows.

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